FDTD STABILITY: CRITICAL TIME INCREMENT

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Abstract

A new approach suitable for determination of the maximal stable time increment for the Finite-Difference Time-Domain (FDTD) algorithm in common curvilinear coordinates, for general mesh shapes and certain types of boundaries is presented. The maximal time increment corresponds to a characteristic value of a Helmholz equation that is solved by a FD method. If this method uses exactly the same discretization as the given FDTD method (same mesh, boundary conditions, order of precision etc.), the maximal stable time increment is obtained from the highest characteristic value. The FD system is solved by an iterative method, which uses only slightly altered original FDTD formulae. The Courant condition yields a stable time increment, but in certain cases the maximum increment is slightly greater([2]).

Keywords

FDTD, stability, critical time increment, critical time step

1. Introduction

Solution of Maxwell's equations in time domain is becoming increasingly important as a tool for microwave component analysis. The precision of electromagnetic field modeling is a complex question. One major factor that has a considerable influence on the precision is the time increment (Δt) of the FDTD algorithm. An optimal value of Δt exists, resulting in fastest and most precise computation. The other reason for setting this constant properly is that even a small excess over its optimal value would result in instability of the algorithm. There is a condition for Δt called Courant ([1]), valid only for rectangular coordinates and an infinitely large mesh. A new approach applicable under more general conditions has been used.

2. Theory

In order to describe the principle of the method, let us consider an example of FDTD for only 2 dimensions:

$$H_z^{n+\frac{1}{2}} = a_h H_z^{n-\frac{1}{2}} - b_h curl_z\left(E_x^n, E_y^n\right) \quad (1h)$$

$$E_x^{n+1} = a_e E_x^n + b_e curl_x\left(H_z^{n+\frac{1}{2}}\right) \quad (1e)$$

$$E_y^{n+1} = a_e E_y^n + b_e curl_y\left(H_z^{n+\frac{1}{2}}\right) \quad (1e')$$

where:

$$a_h = e^{-\frac{\sigma_M}{\mu}\Delta t}, \quad b_h = \frac{1}{\sigma_M}\left(1 - a_h\right) \quad (2h)$$

$$a_e = e^{-\frac{\sigma_E}{\varepsilon}\Delta t}, \quad b_e = \frac{1}{\sigma_E}\left(1 - a_e\right) \quad (2e)$$

These equations require comments:

- Time position of the field samples is marked by the upper suffix; n is integral ($n \in Z$).
- Space differences are not written in full, since the space scheme can be arbitrary. The space scheme is in fact an approximation (discretization) of the continuous *curl* vector operator and in (1) "*curl*" is only a placeholder for the particular discretization.
- It is marked explicitly what component of *curl* approximation is computed (lower suffix) and what it is computed from: e.g. in (1h) the H_z component is updated using the z component of *curl*, which is computed using both E_x^n and E_y^n.
- For sake of simplicity, only a certain field mode is considered: field components remain constant in the direction of z axis.
- The time scheme of (1) together with the constants (2) is applicable for media with significant losses ([1]). Medium properties are described by σ_E, σ_M, ε and μ.

2.1 Time dependence cancellation, formulation of stability

In order to get frequency-domain formulae, let us substitute the following time dependence into (1).

$$E^n = Ee^{j\varphi n} \quad , \quad H^{n+\frac{1}{2}} = He^{j\varphi\left(n+\frac{1}{2}\right)} \tag{3e,3h}$$

where j is the imaginary unit and φ corresponds to angular frequency. Similar procedure can be found in [1].

The formulae follow:

$$\left(e^{+j\frac{1}{2}\varphi} - a_h e^{-j\frac{1}{2}\varphi}\right)\frac{1}{b_h} H_z = -curl_z\left(E_x, E_y\right) \tag{4h}$$

$$\left(e^{+j\frac{1}{2}\varphi} - a_e e^{-j\frac{1}{2}\varphi}\right)\frac{1}{b_e} E_x = +curl_x\left(H_z\right) \tag{4e}$$

$$\left(e^{+j\frac{1}{2}\varphi} - a_e e^{-j\frac{1}{2}\varphi}\right)\frac{1}{b_e} E_y = +curl_y\left(H_z\right) \tag{4e'}$$

The formulae (4) present a set of equations for the unknown φ. The FDTD algorithm (1) is stable, if the following condition is met for all possible φ:

$$\mathrm{Im}\{\varphi\} \geq 0 \qquad \mathrm{Im}\{\varphi\} > 0 \tag{5a,b}$$

In order to be strict, (5b) must be taken into account, in order to ensure stability in all cases. If a limit of stability for Δt, called critical time increment (Δt_c), will be derived using (5a), we will be aware, that at the very limit Δt_c the FDTD may not be stable. For practical cases it has no use, anyway.

We will explain in short why the algorithm (1) can be instable for $\mathrm{Im}\{\varphi\} = 0$ (though the functions (3) are bounded): The functions (3) are in fact solutions of the differential (the discrete case is analogous) equation that describes evolution of the field in time. Its solution is $Ae^{\lambda_1 t} + Be^{\lambda_2 t}$, which is equivalent to (3). The problem is if $\lambda = \lambda_1 = \lambda_2$, as the solution is different: $Ae^{\lambda t} + Bte^{\lambda t}$. We will not discuss this special case any further.

The solutions of (4) must be discussed now.

2.2 Lossless case

First, let us consider lossless case ($\sigma_E = \sigma_M = 0$). The formulae (4) will have the following form:

$$\frac{2}{\Delta t}\sin\left(\tfrac{1}{2}\varphi\right) j\mu H_z = -curl_z\left(E_x, E_y\right) \tag{6h}$$

$$\frac{2}{\Delta t}\sin\left(\tfrac{1}{2}\varphi\right) j\varepsilon E_x = +curl_x\left(H_z\right) \tag{6e}$$

$$\underbrace{\frac{2}{\Delta t}\sin\left(\tfrac{1}{2}\varphi\right)}_{k} j\varepsilon E_y = +curl_y\left(H_z\right) \tag{6e'}$$

A method described in the next chapter is used to obtain the values of k. For the calculations in this chapter it is necessary to know that these values are real ($k \in R$), that there is a finite number of them and that for each k there is also $-k$. Here it is to discuss only the relationship among k, φ and Δt, which is suggested in (6e'):

$$\tfrac{1}{2} k\Delta t = \sin\left(\tfrac{1}{2}\varphi\right) \tag{7}$$

It can be seen that the maximum value of Δt, for which (5a) is true for all the possible values of k, is given by (7) when

$$\varphi = \pi \tag{8}$$

Further, the maximum value of k (k_m) must be substituted:

$$\tfrac{1}{2} k_m \Delta t_c = 1 \tag{9}$$

2.3 Solution of the FD scheme for Helmholz equation

The equation set (6), which is to be solved, is in fact a finite-difference (FD) scheme that can be used to solve a Helmholz equation numerically; the Helmholz equation can be obtained by elimination of E from (6):

$$\underbrace{-\tfrac{1}{\mu} curl_z\left[\tfrac{1}{\varepsilon} curl_x\left(H_z\right), \tfrac{1}{\varepsilon} curl_y\left(H_z\right)\right]}_{\Delta H_z} + k^2 H_z = 0 \tag{10}$$

This form ((10)) can be used to denote both the continuous Helmholz equation and its FD scheme. The latter can be obtained, if the particular discretization is substituted for *curl*. In this case the notation Δ would not stand for the continuous Laplace operator, but for its discrete equivalent.

The solutions of (10) are characteristic functions (called modes), corresponding to characteristic values k.

In [2] a method for solving a discrete system like (10) or (6) is presented. It makes use of the variable separation principle and therefore it can handle only certain boundary conditions. A more general iterative method ([3]) can be used:

The term ΔH_z is evaluated at all the mesh points and these values are used to compute k by means of (10). Because (10) is not satisfied, different values k will be obtained at every mesh point. Rayleigh's formula is used to produce an estimation of k. Then, this estimation together with the values ΔH_z is used to correct the distribution of H using (10). The process is stopped if the change in k is sufficiently small.

In order to obtain the correct k (and hence Δt_c), the FD scheme (10) must be exactly the same as the space scheme of the FDTD method itself (i.e. same spacing of corresponding samples, boundary conditions, order of precision, coordinate system). It is easy to fulfil this condition and moreover it is not necessary to eliminate E

components from (6) as it is suggested in (10). The desired values of ΔH_z can be easily obtained directly from (6) in two steps: First, E is computed from H using (6e,6e'). The values of H computed back from E using (6h) are the values of $(1/k^2)\Delta H_z$. It is simple to change the programmed FDTD (1) and obtain the FD form (6).

Following comments must be mentioned:

- The mesh can have arbitrary shape, but the method was tested only with the simplest boundary conditions – zero elements (be they E or H) at the boundary. And these conditions are assumed in the following text.

- It holds that $k \in R$. One could think at first glance at (6) that complex arithmetic is necessary to be used, but it is not true (consider e.g. the substitution $H = jH'$).

- A discrete mode with the maximal characteristic value k_m exists (unlike the continuous case).

- The material properties ε and μ in (10) (and thus also in (6)) need not be independent on space position (although in this case naming the equation Helmholz may not be justified). Convergence of the method was verified experimentally.

- The discrete mode with the maximal k is distorted so much that it cannot be considered to be an approximation of a continuous mode any more (it is possible only for low k).

Iterations of this method require an initial space distribution of H_z. It shows that the corresponding mode of interest (the one with k_m) has the following property: all the samples neighboring in the directions of all the space coordinates have opposite sign. In order to obtain the mode of interest, the initial distribution has to respect this. In all the experiments the following distribution yielded the desired mode:

$$H_z\big|_{x=i\Delta x+x_0,\, y=i\Delta y+y_0} = (-1)^{i+j} \tag{11}$$

where i,j are integer indices to the field of samples. The formula does not hold for the zero H samples at the boundary (if there are any). The formula is valid for our 2-dimensional example, but it can be extended to 3 dimensions easily.

2.4 Lossy case

Let us mark certain parts of (4) in the similar way it was done in (6e'):

$$\underbrace{\tfrac{1}{b_e}\left(e^{+\frac{j}{2}\varphi} - a_e e^{-\frac{j}{2}\varphi}\right)}_{jk_e} = j\sigma_e \frac{\sin\left(\frac{1}{2}\varphi - \frac{j}{2}\frac{\sigma_e}{\varepsilon}\Delta t\right)}{\sinh\left(\frac{1}{2}\frac{\sigma_e}{\varepsilon}\Delta t\right)} \tag{12e}$$

$$\underbrace{\tfrac{1}{b_h}\left(e^{+\frac{j}{2}\varphi} - a_h e^{-\frac{j}{2}\varphi}\right)}_{jk_h} = \cdots \tag{12h}$$

In the lossless case it was possible to obtain the FD scheme for the Helmholz equation in the form (10), thanks to the fact that k (see (6e')) was constant. In this case, however, k_e, k_h may vary with space position (according to material properties), which would produce a FD scheme with variable and (moreover) non-linear coefficients. Despite these facts an experiment employing a modified method from the previous chapter was found to yield correct results. The only fault is that in this general case we were not able to prove the condition for the limit of stability, which, as it shows, is given by (8), again.

We were able to prove (8) only for the case of constant material properties: In this case k_e and k_h are constant as well and a formula like (10) can be obtained (in (10), ε and μ must be removed and $k = k_h k_e$ substituted). We will show certain steps of the proof only for the special case $\sigma_M/2\mu = \sigma_E/2\varepsilon = p$. The formula, corresponding to (7) in the lossless case, follows:

$$\frac{k^2}{\sigma_E \sigma_M}\sinh^2(p\Delta t) = \sin^2\left(\tfrac{1}{2}\varphi - jp\Delta t\right) \tag{13}$$

The left-hand side of (13) is linearly proportional to k^2, proportional to Δt and is always real and non-negative. The right-hand side can be real, if the imaginary part of φ compensates the term $-jp\Delta t$ or if $\mathrm{Re}\{\varphi\} = \pi$. Growth of the left-hand side (due to a greater Δt or k) can be compensated by growth of $\mathrm{Re}\{\varphi\}$, provided that $\mathrm{Im}\{\frac{1}{2}\varphi\} = jp\Delta t$. It is true, until *sin* reaches its maximum in real domain – after that $\mathrm{Re}\{\varphi\}$ remains equal to π while $\mathrm{Im}\{\varphi\}$ starts to decrease and can become even negative, which means instability.

The formula (13) could be used to determine Δt_c, if (8) and k_m were substituted into it. We used a different method, capable to handle even the fully general case:

Let us write (4) for the stability limit (8) and let us introduce a constant k' in it artificially:

$$k'j\underbrace{\tfrac{1+a_h}{1-a_h}b_h}_{\mu'}H_z = -curl_z\left(E_x, E_y\right) \tag{14h}$$

$$k'j\tfrac{1+a_e}{1-a_e}b_e E_x = +curl_x\left(H_z\right) \tag{14e}$$

$$k'j\underbrace{\tfrac{1+a_e}{1-a_e}b_e}_{\varepsilon'}E_y = +curl_y\left(H_z\right) \tag{14e'}$$

These formulae have exactly the same form as those for the lossless case ((6)). An initial guess of Δt determines the space distribution of ε' and μ' and (14) can be solved for k' by the described method. The artificial constant k' should be 1 in order to get the correct Δt_c. In the next step Δt is corrected and the procedure is repeated until k' is sufficiently close to 1. After an ideal correction of Δt the resulting value of μ' (and similarly ε') should be k'-times greater compared to the same value before the correction. In such an ideal case it is true that if k' is set to 1, then (14) is automatically satisfied, because the correction does not change the set of equations in fact. We would arrive to the solution in one step. It could be done, however, only if the material properties were independent on space position or if the dependence of μ' and ε' on Δt were linear (lossless case). In the general case a change in Δt changes the coefficients in the difference equation (14), therefore more iterations of this method are inevitable.

In the experiment the correction $\Delta t_{i+1} = \Delta t_i k'_i$ was used and the method showed to be convergent. We have not proved this, but the character of the dependence $\mu'(\Delta t)$ or $\varepsilon'(\Delta t)$ promise not to disturb the stability: The derivative of these functions is positive and does not increase as Δt grows. Therefore if Δt changes k'-times ($k' > 0$), the change in μ' (or ε') is smaller than k'-times or is exactly k'-times (if the material in the given space cell is lossless).

2.5 Compact notation of FDTD update formulae

This chapter is added here not only in order to save space during description of the experiment. A very simple, yet exact and well-suited notation, which is especially convenient for theoretical derivations, will be presented. The compact form can be used for abbreviation of the FDTD formulae in rectangular ([1]) and cylindrical ([4]) coordinates and it yields the general form of the coefficients (2) ([1]). FDTD in spherical coordinates, an algorithm of adequate complexity for our experiment, was not found in any literature and therefore it was constructed using this approach.

The compact scheme is a result of expressing Maxwell's equations i) for corresponding coordinate system, ii) in differential form, iii) as a sum of so-called self-adjoined terms:

$$\frac{g}{f}\frac{\partial}{\partial u}(fC) \tag{15}$$

where C is a field component, u stands for a coordinate (u_1, u_2, u_3 or t) and g or f for a function.

It is always possible to write the equations this way, as we can make use of the *curl* operators written by means of the local length units (h_1, h_2, h_3) (see e.g. [5]). An example, first Maxwell's equation for one vector component, follows:

$$\underbrace{\frac{1}{h_2 h_3}\left[\frac{\partial}{\partial u_2}\left(h_3 H_{u_3}\right) - \frac{\partial}{\partial u_3}\left(h_2 H_{u_2}\right)\right]}_{curl_{u_1} H} = \overbrace{\frac{\varepsilon}{e^{\frac{\sigma}{\varepsilon}t}}\frac{\partial}{\partial t}\left(e^{\frac{\sigma}{\varepsilon}t} E_{u_1}\right)}^{\sigma E_{u_1} + \varepsilon \frac{\partial}{\partial t} E_{u_1}} \tag{16}$$

This equation holds for the coordinate u_1 of the coordinate system; – the other 2 equations can be obtained by cyclic interchange of indices *1*,*2*,*3*. The second Maxwell's equation is analogous.

The FDTD is defined by the discretization prescription (or approximation) of (15). Different prescriptions result in different algorithms with corresponding order of precision. The following prescription is considered here:

$$\frac{g}{f}\frac{\partial}{\partial u}(fC) \equiv \frac{g_0}{\frac{1}{\Delta u}\int_I f\,du} \cdot \frac{f_{+\frac{1}{2}} C_{+\frac{1}{2}} - f_{-\frac{1}{2}} C_{-\frac{1}{2}}}{\Delta u} \tag{17}$$

Explanation, further specifications of (17) and comments are required; let us suppose that we want to write the approximation (17) for the point $[t, u_1, u_2, u_3]$:

- The generic coordinate u in (17) can stand for u_1, u_2, u_3 as well as for time t.
- Difference of 2 neighboring field samples, $C_{-\frac{1}{2}}, C_{+\frac{1}{2}}$ is used. The samples are located $-\frac{1}{2}\Delta u$ and $+\frac{1}{2}\Delta u$ (respectively) in the direction u from the point of approximation.
- The difference is weighted by corresponding values of f – samples of f are taken at the points of the samples $C_{-\frac{1}{2}}, C_{+\frac{1}{2}}$.
- The sample of function g (i.e. g_0) is taken at the point of approximation
- The interval, along which the integral is computed, is an oriented abscissa form the point of $C_{-\frac{1}{2}}$ to $C_{+\frac{1}{2}}$.
- It is necessary to respect the fact that the approximation is valid at $[t, u_1, u_2, u_3]$ (which is known as central-difference principle). Approximation of all the terms of Maxwell's equations has to be valid at the same point.

This fact induces formation of the corresponding discretization mesh ([6]).

Let us give an example – discretization of the right-hand side of (16), which yields the general coefficients (2):

Let us rewrite (16), abbreviating its left-hand side as *curl* and expanding only the right-hand side according to (17). (The coordinate u in the general formula (15) stand for t, f for $e^{\frac{\sigma}{\varepsilon}t}$, C for E_{u_1} and g for ε):

$$curl_{u_1}^{n} H = \varepsilon \frac{e^{\frac{\sigma}{\varepsilon}\left(t_n+\frac{1}{2}\Delta t\right)} E_{u_1}^{n+\frac{1}{2}} - e^{\frac{\sigma}{\varepsilon}\left(t_n-\frac{1}{2}\Delta t\right)} E_{u_1}^{n-\frac{1}{2}}}{\Delta t \quad \cdot \quad \frac{1}{\Delta t} \int_{t_n-\frac{1}{2}\Delta t}^{t_n+\frac{1}{2}\Delta t} e^{\frac{\sigma}{\varepsilon}\tau} d\tau} \tag{18}$$

Upper suffix denotes time position of the sample: $E_{u_1}^{n}$ is the sample of E_{u_1} at $t_n = n\Delta t$. The approximation is valid for time instant n, therefore the same must be true for the remaining terms of the Maxwell's equation (which are "hidden" in *curl*) – this is marked by the suffix n at *curl*. The space coordinates remain constant, therefore we avoided to write 3 more space indices in (18).

If the formula (18) were explicit for $E_{u_1}^{n+\frac{1}{2}}$, we would obtain an expression only slightly different from (1e). In order to obtain (1e) exactly, one must make one more substitution into (18) (besides a cosmetic $u_1 \equiv x$). It is self-evident, that field samples in an equation set (like (1), e.g.) must match. The equation (18), as is, would match (1h,1e') only if $(n+\frac{1}{2}) \in Z$. The transition $n \to n+\frac{1}{2}$ makes possible to write $n \in Z$, as it is assumed for (1).

3. Experiment

The method was tested in many experiments. For the experiments in rectangular, cylindrical and spherical coordinates described in [2] it yielded the same results. Both the methods yielded slightly different results compared to the Courant condition [1] (rectangular coordinates). Despite this fact the results are correct. Experiment verified that the Courant condition yields a stable Δt but in certain cases Δt_c is slightly greater ([2]).

An experiment with FDTD in spherical coordinates is described below.

3.1 FDTD update formulae

As an example, we will outline derivation of the update formula for E_r.

The local length units for the spherical coordinates (r, ϑ, φ) are $1, r, r\sin\vartheta$ respectively, hence the left-hand side of (16) follows:

$$\underbrace{\frac{\frac{1}{r}}{\sin\vartheta} \frac{\partial}{\partial\vartheta}\left(H_\varphi \sin\vartheta\right) - \frac{\frac{1}{r\sin\vartheta}}{1} \frac{\partial}{\partial\varphi} H_\vartheta}_{curl_r H} \tag{19}$$

The discretization of (19) can be readily written:

$$curl_r^{n,i,j,k} H = \frac{1}{R^i} \frac{S^{j+\frac{1}{2}} H_\varphi^{n,i,j+\frac{1}{2},k} - S^{j-\frac{1}{2}} H_\varphi^{n,i,j-\frac{1}{2},k}}{\Delta\vartheta \;\cdot\; \Phi^j} -$$

$$- \frac{1}{R^i S^j} \frac{H_\vartheta^{n,i,j,k+\frac{1}{2}} - H_\vartheta^{n,i,j,k-\frac{1}{2}}}{\Delta\varphi} \tag{20}$$

Explanation:

- The upper indices n, i, j, k indicate position of samples: e.g. $H^{n,i,j,k}$ is the value at the point $\left[t_n, r_i, \vartheta_j, \varphi_k\right]$, where $t_n = n\Delta t$, $r_i = i\Delta r$, ...
- R and S stand for samples of the functions r and $\sin\vartheta$ (respectively) at the points given by the suffices. (Those suffices, on which the given function does not depend, are omitted.)

The function Φ^j exactly according to (17) should be the average of $\sin\vartheta$ on the corresponding abscissa, i.e.:

$$\Phi^j = \frac{1}{\Delta\vartheta} \int_{\vartheta_j-\frac{1}{2}\Delta\vartheta}^{\vartheta_j+\frac{1}{2}\Delta\vartheta} \sin\vartheta' \, d\vartheta' \tag{21}$$

At this place we made a simplification in the formulae of the experiment. In all cases the average of '$\sin\vartheta$' was approximated by the value of '$\sin\vartheta$' at the center of the abscissa, i.e. $\Phi^j \equiv S^j$ simply.

In order to get the update formula for E_r, (20) and (18) must be combined. Prior to that, $u_1 \equiv r$ must be substituted into (18) and (18) must be augmented by the left-out space indices.

3.2 Other specifications of the experiment

Simple boundary condition was used: electric walls placed directly to the tangential components of E. The shape of FDTD mesh was a "cube" in spherical coordinates

with the walls at $R_{1;2} = 10;15$, $\Theta_{1;2} = 1;1.5$, $\Phi_{1;2} = 0;1$. The number of cells in the corresponding directions was 15;10;5. In some cases this region was made more complex: In the sub-region that spans 3 cells in *r* direction from the wall at R_1, all the cells from the wall at Θ_1 were cut off by the electric wall to the depth of 3 cells and similarly from the wall Φ_1 to the depth 2. The medium was considered lossless, with $\varepsilon = \mu = 1$, except (in some cases) for the following belt: The *E* components at $r \in \langle R_1 + 3\Delta r, R_1 + 5\Delta r \rangle$ were updated using the constants a_e, b_e (see (2)) corresponding to $\sigma = 1$. Altogether 2×2 cases were investigated: cube or complex region with or without losses.

3.3 Results

In the case of cube with losses $\Delta t_c = 0.289012$ and it was determined by the mode $H_r = 0$, as the time increment obtained from mode $E_r = 0$ was slightly higher: 0.289068. (The indicated precision was obtained in cca 300 iterations without relaxation.) In the case of cube without losses $\Delta t_c = 0.2884806$. In the case of complex region with and without losses $\Delta t_c = 0.290303$ and 0.289759 respectively.

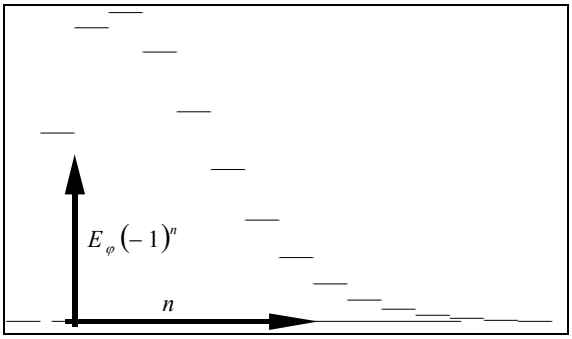

Fig. 1 Discretized field strength of the critical mode plotted against mesh index *n*, along structure radius.

Fig.1 demonstrates a typical waveform of the mode that determines instability. It is a waveform of the component E_φ along the *r* coordinate (with fixed ϑ and φ). The waveform forms an alternating series: the product of adjacent samples is non-positive, but in Fig.1 the rectified waveform was plotted. The zero samples at the boundaries R_1, R_2 are included.

3.4 Remarks to 3 dimensions

In an equation set for 3 dimensions no elimination like (10) is possible in general. Despite that, fortunately, the experiment yielded correct value Δt_c.

During the iterations described in chapter (2.3) the characteristic value is estimated from samples of magnetic field *H*. *H* field is used, because in (6) there is only one component of this field. In general case there will be all 3 components for both *E* and *H*. In our 3-dimensional experiment only the *r* components were used for the purpose. Depending on whether it was E_r or H_r, the experiment yielded 2 characteristic values. The critical time increment was determined by the greater one.

3.5 Verification of results

The obtained critical time increment Δt_c was verified experimentally by means of the FDTD algorithm itself. In order to verify that our critical time increment holds, we made two experiments with Δt slightly over (relative change cca 10^{-6}) and then slightly under our predicted Δt_c. In all cases, exceeding predicted Δt_c values caused instability while lower values proved to be stable.

To be on a safe side, we recomputed the field for long periods to be sure that no other mode does appear.

4. Conclusion

A versatile and easy-to-use method capable of precise determination of the critical time increment for FDTD was presented. The results were verified experimentally for a FDTD in spherical coordinates for non-homogenous medium and non-trivial mesh shapes. The authors assume this method to be new.

Acknowledgements

This research and publication have been sponsored by Czech Grant Agency, contracts no. 102/01/0571 and 102/01/0573, and by Czech Ministry of Education in the frame of project MSM 210000015.

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